Karunya University
(Karunya Institute of Technology and Sciences)
(Declared as Deemed to be University under Sec.3 of the UGC Act, 1965)

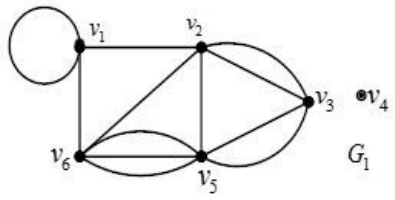
End Semester Examinations - 2015-16 Even Semester - May 2016

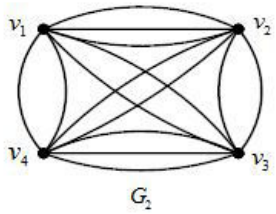
14MA3010 Graph Theory and Algorithms

Set B

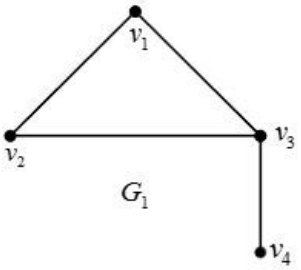
Time : 3 hrs
Total Marks: 100

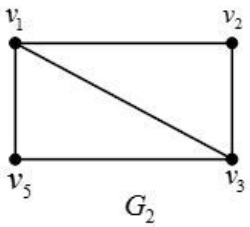
1. a. Find the number of vertices, the number of edges and the degree of each vertices in the given graphs, and establish in each graph the relation between the degree of vertices and the number of edges. (10 Marks)





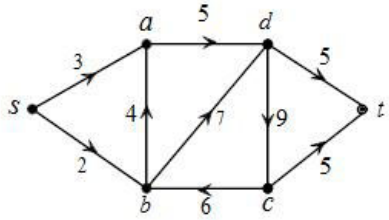
b. Find $G_1 \cup G_2$, $G_1 \cap G_2$, $G_1 \oplus G_2$, $(G_1 \cup G_2) - b$ and fuse vertices b and e in $G_1 \cup G_2$ for the following. (10 Marks)



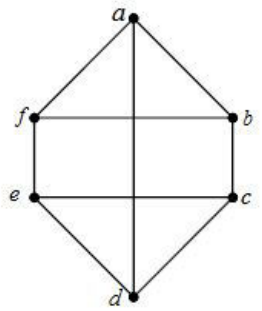


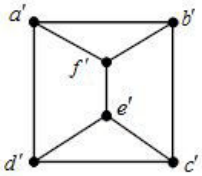
OR

2. a. Find all $s-t$ cuts and their capacities and hence find the maximum flow for the following network. (10 Marks)

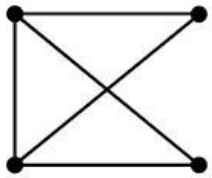


b. Determine whether the following graphs are isomorphic. If the graphs are not isomorphic, give an invariant that the graphs do not share. (5 Marks)

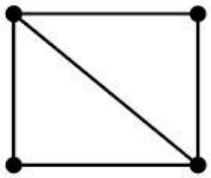




c. For the graphs G_1 and G_2, by labelling the vertices, show that they are isomorphic or why they are not isomorphic. (5 Marks)



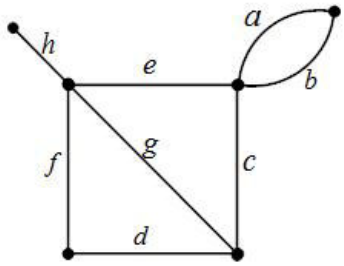
G_1



G_2

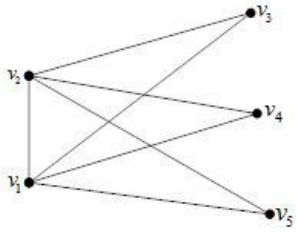
3. a. Prove that every tree has either one or two centers. (10 Marks)

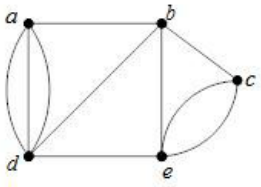
b. Represent the given graph by its cut - set matrix and enumerate five observations on it. (10 Marks)



OR

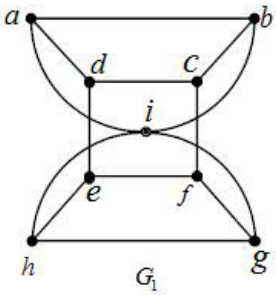
4. a. Determine which of the graphs contain an Eulerian circuit. If it does, then find an Eulerian circuit for the graph. (10 Marks)



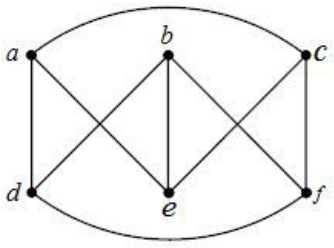


b. Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. (10 Marks)

5. b. Give the planar representation for each of the graphs given. (10 Marks)

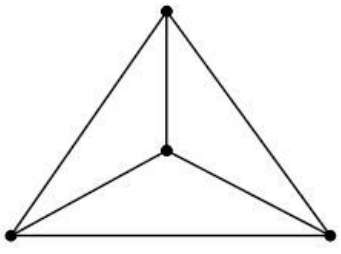


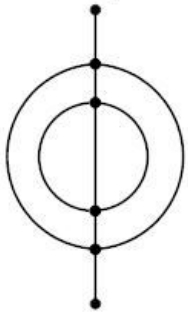
G_1



G_2

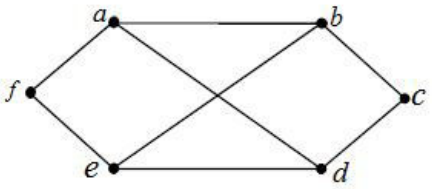
b. Find the number of vertices v, the number of edges e, and the number of regions r, of each of the graphs given below and verify Euler's formula. (10 Marks)

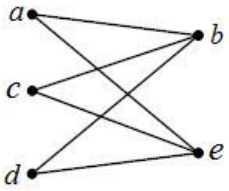




OR

6. a. Prove that a tree with two or more vertices is 2-colourable, and hence find the chromatic number for the graphs. (10 Marks)

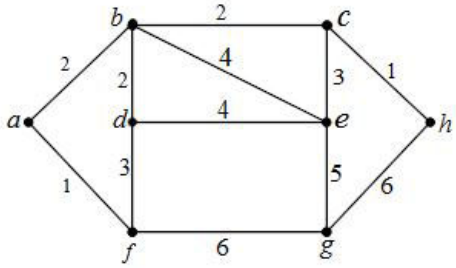




b. Determine, by drawing the graphs, whether the relation represented by the matrices are equivalence relations. (10 Marks)

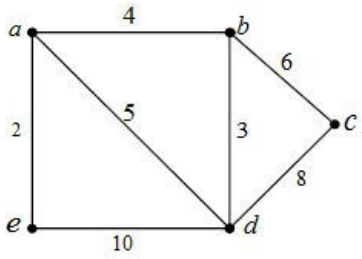
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

7. Using Dijkstra's algorithm find the shortest path between the vertices a to the vertex h for the graph given below. (20 Marks)

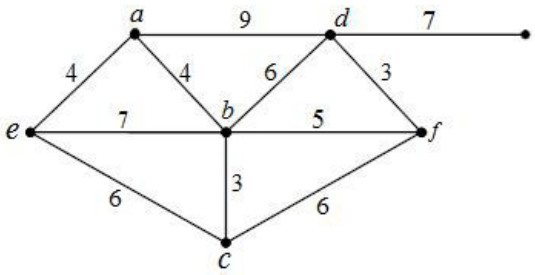


OR

8. a. Write the Kruskal's algorithm and hence find the minimum spanning tree for the given weighted graph. (10 Marks)



b. Write the Prim's algorithm and hence find the minimum spanning tree for the given weighted graph. (10 Marks)



9. a. Solve the following LPP using graphical method.

Max $Z = 3X_1 + 3X_2$, subjected to

$5X_1 + 4X_2 \leq 200$
$3X_1 + 5X_2 \leq 150$
$5X_1 + 4X_2 \geq 100$
$8X_1 + 4X_2 \geq 800$
$X_1, X_2 \geq 0$. (10 Marks)

b. Solve the following LPP using simplex method.

$Max\, Z = 3X_1 + 2X_2$, subjected to

$4X_1 + 3X_2 \leq 12$
$4X_1 + X_2 \leq 8$
$4X_1 - X_2 \leq 8$
$X_1, X_2 \geq 0$. (10 Marks)

Wishing you All the Best
